Closing Tuesday: 2.3 (part 1)
Closing Thursday: 2.3(part 2), 1.5
(**Warning**: Start 2.3 NOW!
BIG assignment.

Quadratic Business Applications

mple: You sell Things.

ts Info: Each Thing costs $6 to produce and you have FC of $20. (slope)

enue Info: The price per Thing is en by a linear (demand) function of antity; You will charge $8 per Thing an order of 7 Things and $20 per ng for an order of 1 Thing.

(a) Find the price function.

(b) Find TR(q), TC(q), profit functions.

PRICE: $p = m(q - q_1) + p_1$

TWO PTS: $(q, p) = (7, 8)$ & $(1, 20)$

$$m = \frac{20-8}{1-7} = \frac{12}{-6} = -2$$

$$p = -2(q - 1) + 20$$

WE CALL THIS THE DEMAND FUNCTION → $\boxed{p = -2q + 22}$

(c) At what quantities is profit zero? (*i.e.* you break even)

(b) $TR(q) = (\text{PRICE})(\text{QUANTITY})$
$= (-2q + 22) \cdot q$

⇒ $\boxed{TR(q) = -2q^2 + 22q}$

★ IMPORTANT MULTIPLY PRICE BY QUANITY TO GET TR FORMULA!

$\boxed{TC(q) = 6q + 20}$

$\text{PROFIT} = TR(q) - TC(q)$
$= [-2q^2 + 22q] - [6q + 20]$
$= -2q^2 + 22q - 6q - 20$

$\boxed{P(q) = -2q^2 + 16q - 20}$

(c) $TR(q) = TC(q)$ OR $P(q) = 0$ (SAME)

$-2q^2 + 22q = 6q + 20$; $-2q^2 + 16q - 20 \stackrel{?}{=} 0$

$$q = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-20)}}{2(-2)}$$

$$q = \frac{-16 \pm \sqrt{96}}{-4}$$

$\frac{-16 - \sqrt{96}}{-4} \approx 1.5505$

$\frac{-16 + \sqrt{96}}{-4} \approx 6.4495$

ROUNDS TO 2 AND 6 THINGS

(d) Find AC(q) and AVC(q).

$$AC(q) = \frac{TC(q)}{q} = \frac{6q+20}{q} = 6 + \frac{20}{q}$$

$$AVC(q) = \frac{VC(q)}{q} = \frac{6q}{q} = 6$$

(e) Find MR(q) and MC(q). ONE ITEM

$$MC(q) = \frac{TC(q+1) - TC(q)}{1}$$

$$= [6(q+1)+20] - [6q+20]$$

$$= 6q + 6 + 20 - 6q - 20$$

$$\boxed{MC(q) = 6}$$

$$MR(q) = \frac{TR(q+1) - TR(q)}{1}$$

$$= [-2(q+1)^2 + 22(q+1)] - [-2q^2 + 22q]$$

$$= -2(q^2+2q+1) + 22q + 22 + 2q^2 - 22q$$

$$= -2q^2 - 4q - 2 + 22 + 2q^2$$

$$\boxed{MR(q) = -4q + 20}$$

(f) At what quantity is profit maximized? What is max profit? What is the selling price when profit is maximum?

OPTION 1 $MR = MC \Rightarrow -4q + 20 = 6$

$\Rightarrow -4q = -14$

$\Rightarrow q = \frac{-14}{-4} = 3.5$

ROUNDS TO $q = 4$ THINGS

OPTION 2 TRY VERTEX OF PROFIT!

$$P(x) = -2q^2 + 16q - 20$$

$$\boxed{q = -\frac{b}{2a} = -\frac{16}{2(-2)} = 4 \text{ THINGS}}$$

$$\text{MAX PROFIT} = P(4) = -2(4)^2 + 16(4) - 20$$

$$= -32 + 64 - 20$$

$$= \boxed{12 \text{ DOLLARS}}$$

CORRESPONDING PRICE $= p = -2q + 22$ (q = 4)

$$= -2(4) + 22$$

$$= \boxed{\$14/\text{THING}}$$

How to approach the problems:

STEP 0: Read the question. Identify the function(s) in the question.

STEP 1: Find/simplify the functions. (Using 2.2 skills)

STEP 2: Roughly sketch the function. Is it a parabola that opens upward or downward? Is it a line with a positive slope or a negative slope?)

STEP 3: Translate the question:

(a) About the shape/vertex?

(b) About solving/quad formula?

(c) A particular business application?

BUSINESS DEFINS

$TR(q) = p \cdot q$

$TC(q) = FC + VC(q)$

$P(q) = TR(q) - TC(q)$

$AC(q) = \frac{TC(q)}{q} \Leftrightarrow TC(q) = q \cdot AC(q)$

$AVC(q) = \frac{VC(q)}{q} \Leftrightarrow VC(q) = q \cdot AVC(q)$

$AR(q) = \frac{TR(q)}{q} \Leftrightarrow TR(q) = q \cdot AR(q)$

$MR(q) = \frac{TR(q + \text{"ONE"}) - TR(q)}{\text{"ONE"}}$

$MC(q) = \frac{TC(q + \text{"ONE"}) - TC(q)}{\text{"ONE"}}$

q = THOUSAND ITEMS ⇒ 0.001 = "ONE"

q = HUNDRED ITEMS ⇒ 0.01 = "ONE"

q = ITEMS ⇒ 1 = "ONE"

Random Problems from Homework:

2.3/2:

$C(x) = 21000 + 55x + 0.3x^2$ and

$R(x) = 425x - 0.7x^2$.

Find break even points. ← DIFFERENT THAN BREAK EVEN PRICE

STEP 0-1: PROFIT = 0

STEP 2: WANT

$$21000 + 55x + 0.3x^2 \overset{?}{=} 425x - 0.7x^2$$

STEP 3 ⇒

$$x^2 - 370x + 21000 \overset{?}{=} 0$$

QUADRATIC FORMULA!

$$x = \frac{370 \pm \sqrt{(-370)^2 - 4(1)(21000)}}{2(1)}$$

$$x = \frac{370 \pm \sqrt{52900}}{2}$$

$$x = \frac{370 \pm 230}{2} \quad \nearrow \frac{370+230}{2} = 300, \quad \searrow \frac{370-230}{2} = 70$$

$$\boxed{x = 70,\ 300}$$

2.3/5: Price per item is $p = 150 - 0.80x$, ← DEMAND FROM A "MONOPOLY MARKET" i.e. YOU GET TO PICK PRICE AND THIS GIVES DEMAND

Find the maximum revenue.

STEP 0-1

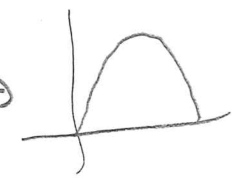
$$TR(x) = \underbrace{(\text{PRICE})}\cdot\underbrace{(\text{QUANTITY})}$$ ★

$$= (150 - 0.8x)x$$

$$\boxed{TR(x) = 150x - 0.8x^2}$$

STEP 2: PARABOLA

SKETCH THIS →



$a = -0.8$

$b = 150$

$c = 0$

STEP 3: TRANSLATE!

MAX ⟺ VERTEX

$$x = -\frac{b}{2a} = -\frac{150}{2(-0.8)} = 93.75 \text{ ITEMS}$$

DID WE ANSWER THE QUESTION?

NO, GIVE MAX TR = ____ DOLLARS!

$$TR(93.75) = 150(93.75) - 0.8(93.75)^2$$

$$= \boxed{\$7031.25}$$

2.3/7(d): For what range of quantities is AVC(q) = (1/30)q²-(3/10)q+1 at most $0.55?

STEP 0-1

$AVC(q) = \frac{1}{30}q^2 - \frac{3}{10}q + 1$

at most \$0.55 ?

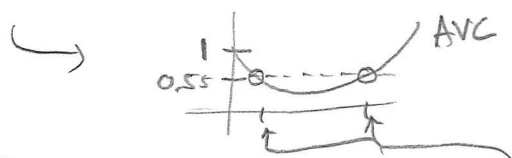
STEP 2 | PARABOLA

$a = 1/30$

$b = -3/10$

$c = 1$

SKETCH!



STEP 3 | AVC is BELOW 0.55 BETWEEN THESE

SOLVE $AVC(q) \overset{?}{=} 0.55$!

$\frac{1}{30}q^2 - \frac{3}{10}q + 1 \overset{?}{=} 0.55$

$\frac{1}{30}q^2 - \frac{3}{10}q + 0.45 \overset{?}{=} 0$

$q = \frac{-(-\frac{3}{10}) \pm \sqrt{(-\frac{3}{10})^2 - 4(\frac{1}{30})(0.45)}}{2(1/30)}$

$= \frac{0.3 \pm \sqrt{0.03}}{0.0\overline{6}}$

$\frac{0.3 + 0.173205081}{0.0\overline{6}} \approx 7.09808 \rightarrow 7.10$ hundred bags (ROUNDING)

$\frac{0.3 - 0.173205081}{0.0\overline{6}} \approx 1.90192 \rightarrow 1.90$ hundred bags

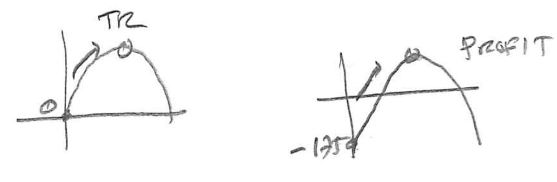
2.3/9(c): Give the longest interval on which TR(q) = -0.16q² + 24q and Profit = (-0.16q² + 24q) – (6q+175) are both increasing.

STEP 0-1 $TR(q) = -0.16q^2 + 24q$ { $a = -0.16$, $b = 24$, $c = 0$ }

$P(q) = -0.16q^2 + 18q - 175$ { $a = -0.16$, $b = 18$, $c = -175$ }

STEP 2 | PARABOLAS!

SKETCH



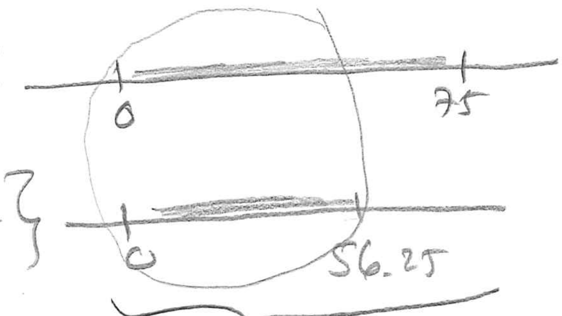
STEP 3 | WANT BOTH INCREASING! THINK VERTEX!

VERTEX OF TR: $x = -\frac{24}{2(-0.16)} \approx 75$

VERTEX OF PROFIT: $x = -\frac{18}{2(-0.16)} \approx 56.25$

TR IS INCREASING FROM x=0 TO x=75

PROFIT IS INCREASING FROM x=0 TO x=56.25



WHEN ARE BOTH TRUE!

0 TO 56.25

2.3/**8,9**: $TR(q) = -0.25q^2 + 30q$

$TC(q) = 17.5q + 100$

(a) Find MR and MC formulas

(b) Find AR and AC formulas

q IN ITEMS

IF IN THOUSANDS THEN 0.001 IS HERE

(a) $MR(q) = \frac{TR(q+1) - TR(q)}{1} = [-0.25(q+1)^2 + 30(q+1)] - [-0.25q^2 + 30q]$

$= -0.25(q^2+2q+1) + 30q + 30 + 0.25q^2 - 30q$

$= -0.25q^2 - 0.5q - 0.25 + 30 + 0.25q^2$

$= \boxed{29.75 - 0.5q}$

$MC(q) = \frac{TC(q+1) - TC(q)}{1} = \frac{(17.5(q+1) + 100) - (17.5q + 100)}{1}$

$= 17.5q + 17.5 + 100 - 17.5q - 100$

$= \boxed{17.5}$

(b) $AR(q) = \frac{TR(q)}{q} = \frac{-0.25q^2 + 30q}{q} = \boxed{-0.25q + 30}$

$AC(q) = \frac{17.5q + 100}{q} = \boxed{17.5 + \frac{100}{q}}$

One More Big Example:
You sell Objects. Your finance specialist analyzed your cost and given you following:

$VC(q) = 0.01q^3 - 0.135q^2 + 0.6075q$

$MC(q) = 0.03q^2 - 0.27q + 0.6075$

FC = 90 hundred dollars

Market price = p = 30 dollars/Object

q in **hundreds of Objects**
VC is in **hundreds of dollars**
MC is in **dollars/Object** (as it always is!)

Question 0:
Find all the other business functions.

TR(q) = PRICE · QUANTITY

$TR(q) = 30q$

TC(q) = FC + VC(q)

$TC(q) = 90 + 0.01q^3 - 0.135q^2 + 0.6075q$

÷ q

$AC(q) = \frac{90}{q} + 0.01q^2 - 0.135q + 0.6075$

÷ q

$AVC(q) = 0.01q^2 - 0.135q + 0.6075$

Profit = P(q) = TR(q) − TC(q)

$= 30q - 90 - 0.01q^3 + 0.135q^2 - 0.6075q$

$= -0.01q^3 + 0.135q^2 + 29.3925q - 90$

Now let's forget the given market price and do a general cost analysis.

What is break even price (BEP)?

LOWEST y-VALUE OF AC OR y-VALUE WHEN AC = MC

$AC(q) = \frac{90}{q} + 0.01q^2 - 0.135q + 0.6075$ OR $\frac{90}{q} + 0.01q^2 - 0.135q + 0.6075 \overset{?}{=} 0.03q^2 - 0.27q + 0.6075$

$\Rightarrow \frac{90}{q} = 0.02q^2 - 0.135q \Rightarrow 90 = 0.02q^3 - 0.135q^2$

$\Rightarrow 0 = 0.02q^3 - 0.135q^2 - 90$

WE DON'T KNOW HOW TO FIND THE LOWEST VALUE FOR THIS IN MATH!!!!

WE DON'T KNOW HOW TO SOLVE IN MATH!!!

SOLVER

$\Rightarrow x \approx 19.094$

y-VALUE = $AC(19.094) \approx 6.38914$

BEP = \$6.39 / ITEM

What is shutdown price (SDP)?

LOWEST y-VALUE OF AVC OR y-VALUE WHEN AVC = MC

$AVC(q) = 0.01q^2 - 0.135q + 0.6075$ OR $0.01q^2 - 0.135q + 0.6075 \overset{?}{=} 0.03q^2 - 0.27q + 0.6075$

QUADRATIC!

$q = -\frac{b}{2a} = -\frac{-0.135}{2(0.01)} = 6.75$

y-VALUE = $AVC(6.75)$

$= 0.01(6.75)^2 - 0.135(6.75) + 0.6075$

≈ 0.151875

SDP = \$0.15 / ITEM

$0 = 0.02q^2 - 0.135q$ ← FACTOR OR QUADRATIC FORMULA

$0 = q(0.02q - 0.135)$

$q = 0$ OR $0.02q - 0.135 = 0$

$q = \frac{0.135}{0.02} = 6.75$

$AC(6.75) \approx 0.151875$

SDP = \$0.15 / ITEM